Initial arrangement.

Elements of Column.	Name of Column.
$v^x l_x$	D_x
$D_x + D_{x+1} + D_{x+2} +$ &c.	N_x
$N_x + N_{x+1} + N_{x+2} +$ &c.	S_x
$v^x(l_{x-1} - l_x) = v^x \delta_x = v^x d_{x-1}$ . . .	C_x
$C_x + C_{x+1} + C_{x+2} +$ &c.	M_x
$M_x + M_{x+1} + M_{x+2} +$ &c.	R_x

In this arrangement, Italic Capitals have been used as the distinctive headings of the columns; but as the D and C columns are the same as Davies's the Roman lettering in these two cases is retained.

Dr. Farr's arrangement.

Elements of Column.	Name of Column.
$v^x l_x$	D_x
$D_x + D_{x+1} + D_{x+2} +$ &c.	N_x
$N_x + N_{x+1} + N_{x+2} +$ &c.	S_x
$v^{x+1}(l_x - l_{x+1}) = v^{x+1}\delta_{x+1} = v^{x+1} d_x$.	$\mathbb{C}_x$
$\mathbb{C}_x + \mathbb{C}_{x+1} + \mathbb{C}_{x+2} +$ &c.	$\mathbb{M}_x$
$\mathbb{M}_x + \mathbb{M}_{x+1} + \mathbb{M}_{x+2} +$ &c. . . .	$\mathbb{R}_x$

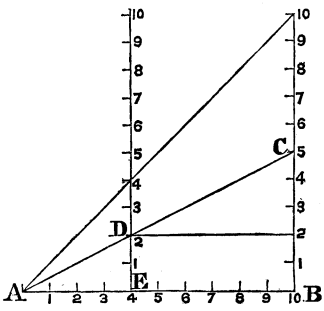
As suggested by Mr. Sprague in last number of the *Journal*, Open Capitals have been used here as the distinctive headings of the columns; but, as the D column is the same as Davies's the Roman lettering for it is still retained. The N and S columns being the same as in the previous arrangement, retain the Italic Capitals as their headings.

Memoir on Instrument for furnishing the D *numbers, to four figures each, in Two Joint Life Annuity Tables, on any basis.* By JARDINE HENRY, *Author of "The Government Annuity Tables," "The Government Life Annuity Commutation Tables," "The Hand Book for Life Assurers," &c. &c.*

[Read before the Institute of Actuaries on 25th November, 1867, and printed in abstract by order of the Council.]

IN this paper, Mr. Henry describes an instrument by the aid of which he has actually obtained the values (to four figures) of many thousand products, being the D values in the various tables he has published. The principle of the instrument, which he claims as

entirely his own invention, may be briefly described as follows:—



Let ABC be a right-angled triangle, of which AB is the base and AC the hypothenuse, and from any point D in the hypothenuse let fall DE perpendicular to AB; then from the similar triangles ADE, ABC, we have

$$DE : AE :: BC : AB$$

$$\text{and } DE = \frac{AE \times BC}{AB}$$

Now if AB be taken as equal to 1, 10, 100, 1000, or 10,000, we may say, disregarding the position of the decimal point, that $DE = AE \times BC$. The instrument used by Mr. Henry consists of a brass right-angled triangle with the two legs equal, each 75 inches long, and having each of the three sides divided into 10,000 equal parts. There is also a moveable T similarly graduated which slides along the triangle, and is to be fixed, where required, parallel to the altitude BC. Then if we require to find the product of two numbers, each consisting of four figures, or less, the one of them is set off on the altitude of the instrument BC, and the line AC is drawn. Then the other number is set off along the base of the instrument, being represented by AE; and the upright T being then fixed at E, and cutting the line AC in D, the length DE is read off on the T, and gives the product of the two given numbers represented by AE and BC.

To keep the T in its place on the face of the Triangle, a copper wire may be passed across the Triangle, permitting the T to slide, but not to fall forward.

The interior of the instrument is filled up with a Zinc plate, and on this is pasted a sheet of paper sufficiently large to cover the whole, and left until it is thoroughly dried.

In practice the T is *clamped* with the screw clamp provided for that purpose—a small square of paper in several folds inserted under the screw of the clamp, and also in the opposite side of the instrument, enabling the clamp to hold firmly.

In order to form a table of joint-life annuities, Mr. Henry sets off on the altitude of the instrument the numbers living at all ages from birth to the extremity of life,—say, for 100 ages, and actually draws the various lines from the angle A to these 100 divisions. The T is then fixed at the point E, such that $AE = l_x v^x$, and reading

off the points where it cuts the various lines drawn, we get a series of products of the form

$$l_y l_x v^x, \quad l_{y+1} l_x v^x, \quad l_{y+2} l_x v^x, \quad \&c.$$

In the same way, setting the T for all the possible values of $l_x v^x$, and reading off the intersections with the same lines, we get all the products which constitute the D numbers in a complete table of two joint lives.

The author proceeds:—These products being taken down on paper, horizontally, furnish a set of numbers which, along with other similar horizontal sets of numbers thus obtained, make the D Columns in Commutation Tables of Joint lives—the addition of them, stage by stage, from the oldest age downwards to the youngest, gives the N Columns.

Supposing that an Instrument can be made of sufficient exactness to show the products correct to the fourth figure of each product—it will be found, on trial, that such numbers, added up, are sufficient to furnish the values of Annuities as exactly as could be done by using five figures and Milne's process, and that for all practical purposes, the latter, and consequently the former also, is as good as Milne's process using 7 figures—in fact, with a medium accuracy in the value of an Annuity to within 2 in the fourth figure of decimals of one pound—one-fifth of one farthing, the error being limited to 1*d.* or 1½*d.*, at most, in any individual case of the value of a Life Annuity of £1 per annum.

As showing, by experiment, the comparative accuracy of taking only four figures for products, a trial has been made with two Tables, calculated by Barrett's method using five figures in the products, which must necessarily render the values exact to three places of Decimals, besides the integers.

These Tables were, Male, Single Life, 3 per cent, and Female, Single Life, 5 per cent, both on the Government Life Annuity basis. The average difference of each value calculated with four figures to the product, was, for both Tables, ·0002, viz., 2 in the fourth place of decimals, or one-fifth of one farthing. For Joint Lives where the values are necessarily diminished compared to Single Lives—the average difference must be less than the above.

Any one acquainted with the amount of difference in the best observations of human mortality will be aware that the differences in such Tables extend to one year's purchase of an annuity, and it may thus be inferred that a greater degree of accuracy than that attained by the Instrument is, practically, useless.

But the important point of the application is the facility with which the products can be read from the Instrument, by one person, so as to be taken down by another person to dictation. This facility exceeds the power of the quickest writer of figures to keep up with—400 products being, however, easily tabulated in an hour—the power attainable being only limited by the power of expression, which may be held to extend to 900 products per hour. To accomplish this last, by the ordinary process of multiplication, would take about ten times that duration, and even the assistance of Logarithms would only enable greater accuracy to be attained than can be had in the ordinary process of multiplying, without any saving of time compared to the latter process.

But, in accuracy, it is much superior to what the results of actual multiplication would bring out.

The last 1500 products of Female Life, Mean Duration of Two Joint Lives, being for the ages from 0 to 16, were furnished and tabulated at the rate of about 500 products per hour.

In speaking of the above I have proceeded on the assumption that the D numbers are taken down from the Instrument in horizontal line, and in ink.

If, however, the mode of taking the D numbers down vertically, and in pencil, be adopted—leaving out the figures that repeat themselves, thus—

Government Tables—Female Life—2 joint Lives—
Interest 4 per cent.
Multiplier D at age 48, 8132, for ages 20, 19, 18, &c.

D numbers.
5844
900
50
6001
42
85
130
62
92
&c.

the rate of rapidity is much increased, the Author having taken down the D numbers at the rate of 2310 D numbers per hour—or one complete Table of D numbers for Two Joint Lives, at one rate of interest, in two hours. The addition of the numbers being necessarily horizontal, in this mode, may create a difficulty which, however, practice may overcome.

Using ink, 2000 D numbers were taken per hour. In stating these facts, the rate of taking down from the Instrument after the

T is set, is alone taken into account. The setting of the T does take a little time which must be deducted in making a practical estimate. But the 500 products per hour referred to as tabulated included the time taken in setting the Instrument, which practice makes very short.

By such means the writer of this Memoir has calculated the mean duration of Life for Two Joint Lives of Males, Females, and their combinations of Male (Elder) and Female Life, and Male (Younger) and Female Life, forming, [with Tables of Mean Duration, and at Interest of 1, 2, 2½, 3, 3½, 4, 5, 6, 7, 8, 9, and 10 per cent for Single Lives,] the first Volume published of *The Government Life Annuity Commutation Tables.* A fourth part of the next volume, viz., Two Joint Lives D and N Columns for Male (Elder) and Female Life at 3, 3½, 4, and 4½ per cent, has also been, as regards D columns, furnished by the Instrument. Specimens at 3 per cent interest are appended—showing the D numbers as actually furnished by the Instrument in antique type and the corrected numbers in ordinary type.

Without the aid of such an Instrument he would not have attempted the work, which, however, the Instrument places within practical compass.

Should it be thought desirable to have MS. Tables, founded on any particular data, such Tables could be furnished by the Instrument with the greatest facility.

In calculating Joint Life *Annuities* it may be here shortly explained that when any particular rate of Interest is wanted—say 3½ per cent per annum, for any particular age or table, say Government Females age 10, 7685, the number alive at age 10, is multiplied by the value of £1. payable ten years hence at 3½ per cent—7089, and the product of the two, or 5448, is the number on the horizontal and hypothenuse scales at which the T is fixed for that particular set—and similarly for the other ages.

As regards the different Instruments which the Author has practically used, these are *two.* The first was formed of Boxwood for the divided parts, and the interior of the triangle was filled up with seasoned Baltic timber, and, after paper had been put on, ruled, and this gave an accuracy, on the average of the products, not differing by more than 4 in the last figure—the size of this Instrument was 100 inches to the side, and was constructed by Messrs. A. Adie & Son.

The second Instrument is formed of Brass Sides, and filled up in the interior with Zinc plate and Paper pasted upon the zinc, on

which (the paper) the lines are ruled, and it is 75 inches to the side. It gives a nearer approach to accuracy, and was also constructed by Messrs. A. Adie & Son.

The error of the products, with reference to the Tables before referred to, has been checked and corrected by using a Table of Multiples to 10 of the original numbers, (*i.e.*, numbers alive) by which means, accuracy to the last figure has been secured.

It is quite possible, the Author believes, to construct a Single Instrument which will accomplish the furnishing of the products correct to the last figure, but as such Instruments have not hitherto been in use to be constructed, there are mechanical difficulties to be overcome.

The Writer may add that, after exhausting the first series of Numbers, running from age 0 to age 76, in the Government Tables of *Females*, which extend from 9754 to 2098, in *Black* lines, he uses *red* lines, with five times the actual numbers, for the Numbers from 77 to 88, which includes the Numbers from 9650 to 2025 (increased five times) *blue* lines for the subsequent series, 89 to 90, taken as they stand, *i.e.* 327·5 and 236·2—then *black* with *intervals*, for a fourth series, 91 to 94, with five times the actual Numbers, *i.e.* 787·5 to 316·2—*red* with *intervals* for a fifth series, 95 to 96, taken as they stand, viz., 35·50 and 23·70—and *blue* with *intervals* for the sixth series, 97 to 100, taken as 5 times the actual numbers, or 98·50 and 39·50, and lastly at age 101, 3·900 taken as it stands, in a blue and black line with intervals, exhausting the Table of Numbers alive, Females.

In reading from the Instrument when the *red* lines are used, as each line represents $\frac{1}{2}^{0}$ times its actual value, the numbers on the T scale must be *doubled*, an operation which practice makes as easy as the reading of the original numbers—the necessity of using five times the number arises from ages after 76 going so far down in the scale as to make the products sink to 3 figures, which is remedied by the process referred to.

It is also expedient occasionally to double the multiplier, and read off the *half* of each result, which practice makes equally easy.

In reading the numbers from the Instrument, a careful eye is required, but a young man with a little experience is quite able to take down the numbers in writing to dictation, or, *vice versâ*, to read the numbers from the Instrument to be taken down by another.

In working the Instrument it is desirable to check the particular age taken down, at intervals of 20 or 30 numbers, as the eye may slip over a particular number (or *line*) and cause misplacement,

which, although it must necessarily be discovered at the end of the particular horizontal set of numbers taken down, is more easily corrected at an earlier stage in the work of taking down the products in a horizontal line.

That the invention is a thoroughly practical one, has been proved to the Author by his own experience, which has embraced calculation, by Logarithms, to the extent of a million and a half of written figures, used in the calculation of *The Government Annuity Tables* published by him in 1859, and the 1st vol. of *The Government Life Annuity Commutation Tables* published last year (1866) extending to 300,000 figures, which would, if wrought by Logarithms, have required 300,000 additional written figures.

As regards the time required to rule the Instrument—it will take about two days. It requires to be done by a person accustomed to accurate ruling, such as the assistant of a Civil Engineer. It is necessary to use, as the ruler, a *straightedge*, or steel square, about 7 or 8 feet long. This last can be obtained from a Civil Engineer. A small magnifying glass is also used, to adjust the straightedge to the point indicating the figure to which the line is to be ruled.

As to the time required to compute or tabulate, a complete set of D numbers—

From 1500 to 2000 D numbers can be dictated and written down horizontally upon paper ruled for the purpose, in one day—and I should say that four days should suffice, provided the parties are not delayed by blunders in dictating, or writing down, the numbers.

The great desideratum in working the Instrument is *accuracy*, because the omission to read a single number (or to observe any one line) puts all the remainder of the numbers of the horizontal series, it may be 90, or 100 numbers, wrong in place. When the Reader and Writer are both accurate, there is nothing more beautiful than the working of the Instrument, from its rapidity and the facility it affords in dealing with large sets of figures.

This is, with reference to the particular Table ruled on the Brass Instrument (Female Life, of the Government Annuity Tables (Henry)), more especially applicable to the black lines, seventy-seven in number. The coloured lines, again, are not so easily worked, because, where a particular adaptation of the Multiplier is needed to avoid carrying the T to a number below 5000 on the horizontal scale of the Instrument, and we double or halve the Multiplier for that purpose—there is then some trouble in working

the coloured lines. Supposing we *double* the multiplier, then we must halve the numbers applicable to the black lines, but the red lines having been ruled to numbers 5 times the amount of the proper numbers in the Table (practically *halved*), then the red lines in our supposed case *read natural*—that is, are taken down as they stand. The blue lines again, having been ruled to the true numbers, the numbers on the T scale marked by them are halved, and so on alternately to the end. When the Multiplier is *halved*, then the black line numbers on the T require to be doubled, while the red line numbers would require to be quadrupled, the blue line numbers doubled, and so on alternately. This, however, can generally be avoided, as is desirable, the quadrupling of numbers being a work which obliges recourse to be had to pencil and paper for correct results.

As observed before, the difficulties are confined to the numbers above age 76, that is, a fourth only of the whole, and by judicious management can generally be avoided.

In order to show the working of the Instrument, I shall, after the Tables showing the Numbers alive at every age, give a set of products derived from the Brass Instrument—then the corrected numbers, being those intended for being printed—after disposing thus of the D numbers, the resulting N numbers for the uncorrected and corrected D numbers, will be shown for every tenth division or part of the whole—finally the money values applicable to each—being Male (Elder) and Female Life, 3 per cent.

Male Life.

Number who complete the age opposite to each.							
Age of A.	Number Alive. *a.*	Age of A.	Number Alive. *a.*	Age of A.	Number Alive. *a.*	Age of A.	Number Alive. *a.*
0	100647	13	75482	26	65086	40	54210
1	84610	14	75025	27	64127	41	53372
2	81828	15	74504	28	63197	42	52639
3	80614	16	74175	29	62343	43	51882
4	80156	17	73687	30	61600	44	51134
5	79641	18	72903	31	60836	45	50394
6	79097	19	72043	32	60010	46	49762
7	78480	20	71202	33	59524	47	49166
8	77910	21	70223	34	58539	48	48558
9	77417	22	69136	35	57955	49	47893
10	76782	23	68038	36	57351	50	47073
11	76224	24	66913	37	56622	51	46206
12	75856	25	65948	38	55829	52	45276
				39	55057		

Male Life (continued).

Number who complete the age opposite to each.							
Age of A.	Number Alive. *a.*	Age of A.	Number Alive. *a.*	Age of A.	Number Alive. *a.*	Age of A.	Number Alive. *a.*
53	44223	65	30101	76	13707	87	2353
54	43138	66	28829	77	12422	88	1834
55	42115	67	27374	78	11164	89	1452
56	41091	68	25957	79	9973		
57	40001	69	24331			90	1084
58	38942			80	8736	91	752
59	37870	70	22574	81	7489	92	474
		71	20875	82	6303	93	406
60	36722	72	19380	83	5323	94	338
61	35441	73	17828	84	4597		
62	34108	74	16276			95	188
63	32831			85	3895	96	94
64	31393	75	14947	86	3070		

NOTE.—The number of Male Children newly born was derived from the probability of a Male Child newly born surviving one year ($_1 0$) as given by the English Life Table, No. 1.

Female Life.

Number who complete the age opposite to each.							
Age of A.	Number Alive. *a.*	Age of A.	Number Alive. *a.*	Age of A.	Number Alive. *a.*	Age of A.	Number Alive. *a.*
0	97544	26	68271	52	50337	78	17454
1	84610	27	67606	53	49555	79	15753
2	83490	28	66965	54	48736	80	14280
3	82020	29	66334	55	47867	81	12438
4	80563	30	65745	56	46922	82	10730
5	79739	31	65083	57	46041	83	9354
6	79013	32	64309	58	45113	84	7965
7	78230	33	63570	59	44151	85	6842
8	77647	34	62911	60	43316	86	5876
9	77204	35	62347	61	42361	87	4900
10	76853	36	61791	62	41233	88	4050
11	76538	37	61174	63	40177	89	3275
12	76216	38	60450	64	39012	90	2362
13	75860	39	59708	65	37759	91	1575
14	75421	40	59036	66	36523	92	1050
15	74907	41	58270	67	35232	93	869
16	74342	42	57584	68	33907	94	632
17	73755	43	56966	69	32432	95	355
18	73157	44	56319	70	30876	96	237
19	72518	45	55698	71	29331	97	197
20	71875	46	54982	72	27740	98	158
21	71271	47	54254	73	26087	99	118
22	70678	48	53437	74	24419	100	79
23	70076	49	52551	75	22699	101	39
24	69500	50	51703	76	20984		
25	68912	51	50976	77	19300		

NOTE.—The number of Female Children newly born (of age 0), is derived from the English Life Table, No. 1.

Government Life Annuity Commutation Tables.
Male (Elder) and Female Life.

ANNUITY ON TWO JOINT LIVES AT 3 PER CENT.

Difference of Age 12.

Age of Elder.	FROM BRASS INSTRUMENT.		Uncorrected.	Corrected.	1+value of Annuity of £1. BRASS INSTRUMENT.	
	Uncorrected. D.	Corrected. D.	N.	N.	Uncorrected.	Corrected.
96	4388	4384				
95	1060	1060				
94	2252	2252				
93	3234	3232				
92	4462	4462				
91	8032	8038				
90	1324	1324	3271	3272	2·471	2·471
89	2019	2019				
88	2854	2854				
87	4076	4081				
86	5885	5900				
85	8240	8236				
84	1065	1066				
83	1342	1342				
82	1728	1724				
81	2215	2215				
80	2784	2784	11769	11767	4·227	4·227
79	3398	3401				
78	4068	4065				
77	4816	4817				
76	5652	5658				
75	6546	6542				
74	7526	7531				
73	8731	8731				
72	9992	9994				
71	1132	1130				
70	1286	1286	8665	8666	6·738	6·739
69	1457	1457				
68	1632	1632				
67	1808	1803				
66	1997	1997				
65	2185	2185				
64	2384	2384				
63	2601	2604				
62	2824	2822				
61	3070	3070				
60	3331	3331	31954	31956	9·593	9·594
59	3592	3592				
58	3853	3855				
57	4133	4133				
56	4420	4420				
55	4723	4723				

Government Life Annuity Commutation Tables.
Male (Elder) and Female Life.

ANNUITY ON TWO JOINT LIVES AT 3 PER CENT.

Difference of Age 12 (*continued*).

Age of Elder.	FROM BRASS INSTRUMENT.		Uncorrected.	Corrected.	1+value of Annuity of £1. BRASS INSTRUMENT.	
	Uncorrected. D.	Corrected. D.	N.	N.	Uncorrected.	Corrected.
54	5035	5035				
53	5379	5379				
52	5752	5749				
51	6110	6108				
50	6498	6492	81449	81442	12·534	12·545
49	6870	6881				
48	7254	7261				
47	7646	7644				
46	8036	8040				
45	8460	8467				
44	8958	8958				
43	9474	9474				
42	1000	1000				
41	1054	1054				
40	1113	1113	16981	16983	15·257	15·258
39	1175	1175				
38	1240	1240				
37	1307	1307				
36	1376	1376				
35	1443	1443				
34	1515	1515				
33	1592	1592				
32	1675	1675				
31	1765	1765				
30	1858	1858	31927	31929	17·183	17·185
29	1950	1950				
28	2052	2052				
27	2160	2162				
26	2274	2277				
25	2388	2390				
24	2508	2510				
23	2640	2640				
22	2771	2773				
21	2913	2913				
20	3062	3062	56645	56658	18·499	18·503
19	3213	3213				
18	3384	3384				
17	3558	3556				
16	3725	3723				
15	3923	3923				
14	4141	4141				
13	4346	4349	82935	82947	19·083	19·073
12	5183	5192	88118	88139	17·001	16·976

Government Life Annuity Commutation Tables.
Male (Elder) and Female Life.

ANNUITY ON TWO JOINT LIVES AT 3 PER CENT.

Difference of Age 24.

Age of Elder.	From Brass Instrument.		Uncorrected.	Corrected.	1+value of Annuity of £1. Brass Instrument.	
	Uncorrected. D.	Corrected. D.	N.	N.	Uncorrected.	Corrected.
96	1532	1527				
95	3330	3326				
94	6478	6485				
93	8430	8426				
92	1060	1060				
91	1798	1798				
90	2767	2767	7602	7601	2·747	2·747
89	3954	3950				
88	5302	5309				
87	7222	7225				
86	9955	9961				
85	1335	1337				
84	1662	1664				
83	2020	2020				
82	2520	2518				
81	3143	3146				
80	3851	3851	17934	17940	4·657	4·659
79	4620	4620				
78	5426	5426				
77	6318	6318				
76	7298	7300				
75	8308	8305				
74	9442	9440				
73	1083	1083				
72	1233	1233				
71	1391	1389				
70	1568	1568	11209	11207	7·149	7·147
69	1763	1763				
68	1958	1958				
67	2154	2152				
66	2359	2359				
65	2570	2570				
64	2797	2797				
63	3045	3045				
62	3301	3301				
61	3570	3573				
60	3852	3852	38578	38577	10·020	10·015
59	4128	4128				
58	4408	4411				
57	4717	4717				
56	5050	5048				
55	5394	5394				

Government Life Annuity Commutation Tables.
Male (Elder) and Female Life.

ANNUITY ON TWO JOINT LIVES AT 3 PER CENT.

Difference of Age 24 (*continued*).

Age of Elder.	From Brass Instrument.		Uncorrected.	Corrected.	1+value of Annuity of £1. Brass Instrument.	
	Uncorrected. D.	Corrected. D.	N.	N.	Uncorrected.	Corrected.
54	5750	5750				
53	6124	6124				
52	6522	6521				
51	6914	6917				
50	7326	7332	94911	94919	12·955	12·946
49	7744	7753				
48	8164	8167				
47	8592	8590				
46	9032	9030				
45	9484	9495				
44	1001	1001				
43	1056	1056				
42	1113	1113				
41	1172	1172				
40	1234	1234	19368	19371	15·695	15·685
39	1301	1301				
38	1369	1369				
37	1439	1439				
36	1509	1509				
35	1578	1577				
34	1647	1647				
33	1724	1725				
32	1808	1808				
31	1903	1903				
30	2006	2005	35652	35654	17·782	17·783
29	2111	2111				
28	2224	2224				
27	2366	2368				
26	2517	2520	44870	44877	17·827	17·808
25	2672	2665				
24	3206	3211	50748	50753	15·829	15·806

From the specimens given, which have been taken at random, and may thus be held as a fair representation of the whole, it will be seen that the Brass Instrument's figures, taken without correction are very close to the corrected results, and, as regards the actual values of annuities, derived from either, there is no appreciable difference, excepting at the two last values of annuities, where it rises to 6*d.*

As regards Table of Diff. of Age 12. The greatest difference is at ages 12 M., 0 F., and is ·025 *plus* on Instrument. This arises mainly from the line on the Brass Instrument ruled to the age ·0, representing the number of Females born, being the worst ruled of all.

This appears from the next age, 13 M., 1 F., when the difference is only ·010 plus on Instrument, being the greatest difference except the above, the average of error being ·0015 plus, or, leaving out the last, ·001 plus, equal to $\frac{1}{4}d.$ on each value, or a little less.

On trying the next Table for ages (older) 90, 80, 70, 60, 50, 40, 30, 26, and 24, the greatest difference appears at 24, being ·023 plus on Instrument, and the average is ·007 plus on Instrument, or about $1\frac{1}{2}d.$

From having made the same calculations by the *Wood* Instrument, which had not the advantage of the Brass Instrument in the coloured lines and increased numbers for these, but was ruled uniformly to the bottom of the Triangle, I am enabled to state that the average error in the first Table is ·023, and the greatest error ·088, both plus. In the Second Table, the average error was ·012, and the greatest error ·088, both minus.

This shows, that even with the imperfection of the Wood Instrument, a tolerable approach was made to accuracy. Had the coloured lines been extended to the Wood Instrument, thereby obtaining four figures in every result, (instead of three at the lower lines,) the approach to accuracy would have been much nearer.

As the matter stands, I think it established that an Instrument could be constructed and ruled, so as to give accurately the four figures required, and thus supersede, in the construction of Two Joint Life Tables, the employment of multiplication, or of logarithms.

If it were practicable to add the numbers side by side, the taking down being made so easy, the calculation would be much lighter, it having been stated, as before, that it is perfectly practicable to take down the D numbers at the rate of 2300 of these in an hour.

The time required to shift the T to each Multiplier has not however been taken into account. In taking down 2300 numbers, we would have about 46 T sets of figures, *i.e.* the average number read from each setting of the T being about 50, there would thus be 46 settings of the T—and the time required to set and clamp the T at the bottom and top would take probably two minutes each setting. The actual quantity of products obtained would thus be

diminished to 910 products per hour, on the average. Still at this rate a complete set of D numbers for Two Joint Lives of Males or Females, or these combined, could be written down in less than six hours.

Eighth Annual Report of the Superintendent of the Insurance Department—State of New York.

ALTHOUGH the general principles upon which the business of Insurance is based, must be the same wherever it is carried on; yet its particular developments must of necessity vary in countries under different laws, and with a public opinion differently educated. An investigation therefore of the book before us, with the view of pointing out those points wherein the practice of our Trans-Atlantic Brethren differs from ours, may not be without advantage to the interests of Insurance in this country.

The first point which attracts our attention, is the public advantage which results in the United States, from the publication in one volume by an accredited State Officer of the accounts of all the different Insurance Companies doing business in the State. In this way is obtained a clear comparative view of the working of each, and the improvements of some Offices and the shortcomings of others are authoritatively submitted to the public gaze; and thus no doubt a great check is given to the founding of bubble Companies, and to the continued existence of those reared upon an insufficient basis.

The appointment of a Government Inspector with power to order Insurance Companies to publish Balance Sheets, may appear at first sight an undue infringement of the liberty of the subject, and an interference with the freedom of trade; but we think it will appear upon investigation that such an appointment in this country with suitable limitations would be productive of much good, and tend greatly to the protection of the public. There might, it is true, be instances where the compulsory publication of accounts would injure and perhaps destroy companies recently founded, which otherwise might have struggled on for years with a greater or less degree of vitality; but we think that the good effects would immensely preponderate. We are of opinion, however, that the functions of such an Officer should be strictly limited to obtaining and making public a true statement of the business transactions and the financial position of each Company.